



Model Question Paper HSSC-II

Mathematics

(2nd Set) SOLUTION

Section – A

Q1.

1	C	2	B	3	C	4	B	5	A	6	B	7	D	8	B	9	A	10	A
11	B	12	D	13	C	14	B	15	B	16	D	17	B	18	A	19	B	20	D

Section – B

Q2.

(i)
$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos 5x}{1 - \cos 7x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{5x}{2}\right)}{2 \sin^2\left(\frac{7x}{2}\right)} = \lim_{x \rightarrow 0} \left(\frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{7x}{2}\right)} \right)^2$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin\left(\frac{5x}{2}\right)}{5x/2} \times \frac{7x/2}{\sin\left(\frac{7x}{2}\right)} \times \frac{5x/2}{7x/2} \right)^2$$

$$= \frac{25}{49} \times \left[\lim_{x \rightarrow 0} \left(\frac{\sin\left(\frac{5x}{2}\right)}{\frac{5x}{2}} \right) \right]^2 \times \left[\lim_{x \rightarrow 0} \left(\frac{7x/2}{\sin\left(\frac{7x}{2}\right)} \right) \right]^2$$

$$= \frac{25}{49} \times (1)^2 \times (1)^2 = \frac{25}{49} \quad \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

(ii)
$$S(x) = \frac{15x^3 + 23x + 45}{x^2 - 3x + 25}$$

(a)
$$\lim_{x \rightarrow 3} S(x) = ?$$

$$\lim_{x \rightarrow 3} S(x) = \lim_{x \rightarrow 3} \left(\frac{15x^3 + 23x + 45}{x^2 - 3x + 25} \right) = \frac{15(3)^3 + 23(3) + 45}{3^2 - 3(3) + 25} = 20.76$$

(b)
$$\lim_{x \rightarrow 15} S(x) = ?$$

$$\lim_{x \rightarrow 15} S(x) = \lim_{x \rightarrow 15} \left(\frac{15x^3 + 23x + 45}{x^2 - 3x + 25} \right) = \frac{15(15)^3 + 23(15) + 45}{(15)^2 - 3(15) + 25} \lim_{x \rightarrow 15} S(x) = 248.85$$

(iii)
$$y = (2x - 3)^{-5}$$

Taking increment to both sides

$$y + \delta y = [2(x + \delta x) - 3]^{-5}$$

Subtracting above equations as

$$\begin{aligned}
y + \delta y - y &= [2(x + \delta x) - 3]^{-5} - (2x - 3)^{-5} \\
\delta y &= [2x + 2\delta x - 3]^{-5} - (2x - 3)^{-5} \\
\delta y &= [(2x - 3) + 2\delta x]^{-5} - (2x - 3)^{-5} \\
\delta y &= (2x - 3)^{-5} \left[1 + \frac{2\delta x}{2x - 3} \right]^{-5} - (2x - 3)^{-5} \\
\delta y &= (2x - 3)^{-5} \left[\left(1 + \frac{2\delta x}{2x - 3} \right)^{-5} - 1 \right] \\
\delta y &= (2x - 3)^{-5} \left[\left\{ 1 + (-5) \left(\frac{2\delta x}{2x - 3} \right) + \frac{(-5)(-6)}{2!} \left(\frac{2\delta x}{2x - 3} \right)^2 + \dots \right\} - 1 \right] \\
\delta y &= (2x - 3)^{-5} \left[(-5) \left(\frac{2\delta x}{2x - 3} \right) + \frac{(-5)(-6)}{2!} \left(\frac{2\delta x}{2x - 3} \right)^2 + \dots \right] \\
\delta y &= (2x - 3)^{-5} (-5) \left(\frac{2\delta x}{2x - 3} \right) \left[1 + \frac{(-6)}{2!} \left(\frac{2\delta x}{2x - 3} \right) + \dots \right] \\
\frac{\delta y}{\delta x} &= -10(2x - 3)^{-6} \left[1 + \frac{(-6)}{2!} \left(\frac{2\delta x}{2x - 3} \right) + \dots \right] \\
\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= -10(2x - 3)^{-6} \lim_{\delta x \rightarrow 0} \left[1 + \frac{(-6)}{2!} \left(\frac{2\delta x}{2x - 3} \right) + \dots \right] \\
\frac{dy}{dx} &= -10(2x - 3)^{-6} [1 + 0 + \dots] \\
\frac{dy}{dx} &= -10(2x - 3)^{-6}
\end{aligned}$$

(iv) $f(x) = \sin 2x + 2\cos x$

Differentiating w.r.t. x

$$f'(x) = 2\cos 2x - 2\sin x \quad ; \quad f''(x) = -4\sin 2x - 2\cos x$$

For extreme values put $f'(x) = 0$

$$2\cos 2x - 2\sin x = 0$$

$$\cos 2x - \sin x = 0$$

$$1 - 2\sin^2 x - \sin x = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(\sin x + 1)(2\sin x - 1) = 0$$

$$\sin x + 1 = 0 \text{ or } 2\sin x - 1 = 0$$

$$\sin x = 1 \text{ or } \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2} \text{ or } x = \frac{\pi}{6}$$

$$f''\left(\frac{\pi}{2}\right) = -4\sin \pi - 2\cos \frac{\pi}{2} = 0$$

$$f''\left(\frac{\pi}{6}\right) = -4\sin\left(\frac{\pi}{3}\right) - 2\cos\left(\frac{\pi}{6}\right) = -3\sqrt{3} < 0$$

f shows its maximum value at $x = \frac{\pi}{6}$

$$f_{\max}: f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{2\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

(v) $y = \ln \left[\tan^{-1} \left(\frac{2x}{1+x^2} \right) \right]$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \left[\tan^{-1} \left(\frac{2x}{1+x^2} \right) \right]^{-1} \frac{d}{dx} \left[\tan^{-1} \left(\frac{2x}{1+x^2} \right) \right]$$

$$\frac{dy}{dx} = \left[\tan^{-1} \left(\frac{2x}{1+x^2} \right) \right]^{-1} \frac{1}{1 + \left(\frac{2x}{1+x^2} \right)^2} \frac{d}{dx} \left[\left(\frac{2x}{1+x^2} \right) \right]$$

$$\frac{dy}{dx} = \left[\tan^{-1} \left(\frac{2x}{1+x^2} \right) \right]^{-1} \frac{(1+x^2)^2}{(1+x^2)^2 + 4x^2} \left[\frac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2} \right]$$

$$\frac{dy}{dx} = \left[\tan^{-1} \left(\frac{2x}{1+x^2} \right) \right]^{-1} \frac{2-2x^2}{(1+x^2)^2 + 4x^2}$$

(vi) $\int_{-3}^4 [2x + |x + 1|] dx$
 $= \int_{-3}^{-1} [2x - (x + 1)] dx + \int_{-1}^4 [2x + (x + 1)] dx$
 $= \int_{-3}^{-1} [x - 1] dx + \int_{-1}^4 [3x + 1] dx$
 $= \left[\frac{x^2}{2} - x \right]_{-3}^{-1} + \left[\frac{3x^2}{2} + x \right]_{-1}^4$
 $= \left[\left(\frac{1}{2} - 1 \right) - \left(\frac{9}{2} - 3 \right) \right] + \left[\left(\frac{48}{2} + 4 \right) - \left(\frac{3}{2} - 1 \right) \right]$
 $= \left[\frac{3}{2} - \frac{15}{2} \right] + \left[\frac{56}{2} - \frac{1}{2} \right] = \frac{43}{2}$

(vii) $I = \int \sin^5 x dx$
 $I = \int (\sin x)(\sin x)^4 dx = \int (\sin x)(\sin^2 x)^2 dx = \int (1 - \cos^2 x)^2 (\sin x) dx$
 Let $y = \cos x \Rightarrow dy = -\sin x dx \Rightarrow -dy = \sin x dx$
 $I = - \int (1 - y^2)^2 dy = - \int (y^4 - 2y^2 + 1) dy$
 $I = - \frac{y^5}{5} + \frac{2y^3}{3} - y + C$
 $I = - \frac{y^5}{5} + \frac{2y^3}{3} - y + C$
 $I = - \frac{\cos^5 y}{5} + \frac{2 \cos^3 y}{3} - \cos y + C$

(viii) $I = \int x\sqrt{x+2} dx$
 Let $t = x + 2 \Rightarrow x = t - 2 \Rightarrow dx = dt$
 $I = \int (t - 2)\sqrt{t} dt$
 $I = \int (t^{3/2} - 2t^{1/2}) dt$
 $I = \frac{t^{5/2}}{5/2} - \frac{2t^{3/2}}{3/2} + C$
 $I = \frac{2}{5}(x + 2)^{5/2} - \frac{4}{3}(x + 2)^{3/2} + C$

(ix) (a) $l_1: y = 2x \quad l_2: 2x + y - 12 = 0 \quad l_3: y = 2$

Solving l_1 & l_2

Substitute l_1 in l_2

$$2x + 2x - 12 \Rightarrow 4x = 12 \Rightarrow x = 3$$

Now substitute $x = 3$ in l_1 , $y = 2(3) = 6$

l_1 & l_2 intersect at A(3,6)

Solving l_2 & l_3

Substitute l_3 in l_2 we get

$$2x + 2 - 12 = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5$$

Substitute $x = 5$ in l_1 , $y = 2(5) = 10$

l_2 & l_3 intersect at B(5,10)

Solving l_1 & l_3

Substitute l_3 in l_1 $2 = 2x \Rightarrow x = 1$

l_1 & l_3 intersect at C(1,2)

Hence A(3,6), B(5,10), C(1,2) are the vertices of the triangular region.

(b) Area of the triangular region having vertices

$(x_1, y_1) = A(3,6)$, $(x_2, y_2) = B(5,10)$, $(x_3, y_3) = C(1,2)$ is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 3 & 6 & 1 \\ 5 & 10 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2} [3(10 - 2) - 6(5 - 1) + 1(10 - 10)]$$

$$\Delta = \frac{1}{2} [24 - 24 + 0] = 0$$

(x) Let $l_1: 3x - 2y + 4 = 0$ be the given line.

$$\text{Slope of } l_1: m_1 = -\frac{3}{-2} = \frac{3}{2}$$

Let l_2 be the line perpendicular to l_1

$$\text{Slope of } l_2: m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3}$$

Equation of l_2 with $m_2 = -\frac{2}{3}$ and through $(x_1, y_1) = (0, 0)$ is

$$y - y_1 = m_2(x - x_1)$$

$$y - 0 = -\frac{2}{3}(x - 0)$$

$$3y = -2x$$

$$l_2: 2x + 3y = 0$$

Finding point of intersection of l_1 , l_2 by Cross Multiplication method

$$l_1: 3x - 2y + 4 = 0$$

$$l_2: 2x - 3y + 0 = 0$$

$$\frac{x}{0+12} = \frac{-y}{0-8} = \frac{1}{-9+4}$$

$$\frac{x}{12} = \frac{y}{8} = \frac{1}{-5}$$

$$x = \frac{-12}{5} \quad \text{and} \quad y = -\frac{8}{5}$$

Hence $\left(\frac{-12}{5}, -\frac{8}{5}\right)$ is the point of intersection of l_1 , l_2

(xi) Line: $3x - y = 16$ or $y = 3x - 16$ eqn-1

Circle: $3x^2 + 3y^2 - 12x - 15y - 45 = 0$ eqn-2

Substituting value of y form eqn-1 in eqn-2 to obtain

$$3x^2 + 3(3x - 16)^2 - 12x - 15(3x - 16) - 45 = 0$$

$$3x^2 + 27x^2 - 288x + 768 - 12x - 45x + 240 - 45 = 0$$

$$30x^2 - 345x + 963 = 0$$

$$10x^2 - 115x + 321 = 0$$

By using the Quadratic formula

$$x = \frac{115 \pm \sqrt{(-115)^2 - 4(10)(321)}}{2(10)} = \frac{115 \pm \sqrt{13225 - 12840}}{20} = \frac{115 \pm \sqrt{385}}{20}$$

$$x = 4.77 \text{ or } 6.73$$

Now, we substitute these values of x in eqn-2, we get

$$y = 3(4.77) - 16 \quad ; \quad y = 3(6.73) - 16$$

$$y = -1.69 \quad ; \quad y = 4.19$$

Hence $(4.77, -1.69)$ and $(6.73, 4.19)$ are the points of intersection of the given line and circle.

(xii) Equation of ellipse: $9x^2 + 13y^2 = 117$

Dividing by 117 $\frac{x^2}{13} + \frac{y^2}{9} = 1$

Compare with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

we get $a^2 = 13$ and $b^2 = 9$

for eccentricity $b^2 = a^2(1 - e^2)$

$$9 = 13(1 - e^2)$$

$$e = \frac{2}{\sqrt{13}}$$

Distance between the directrices $= \frac{2a}{e} = \frac{2\sqrt{13}}{\frac{2}{\sqrt{13}}} = 13$

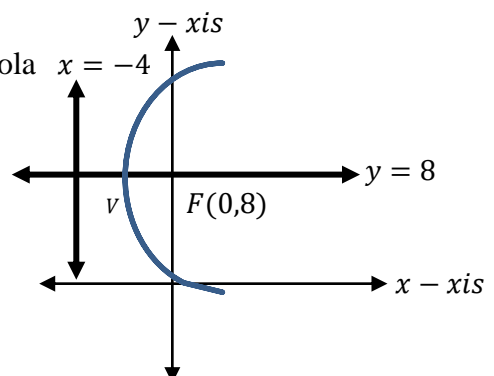
(xiii) Let $p: (y - k)^2 = 4a(x - h)$ be the required parabola

with vertex $V(h, k)$, axis of symmetry $y = 8$

focus $F(0, 8)$ and directrix $x = -4$

Since vertex lies in the mid of directrix and focus on the axis of symmetry.

Thus $V(h, k) = V(-2, 8) \Rightarrow h = -2, k = 8$



$$\text{Also } a = |VF| = \sqrt{(-2-0)^2 + (8-8)^2} = 2$$

Substituting values in eqn-p

$$(y-8)^2 = 4(2)(x+2)$$

Hence $(y-8)^2 = 8(x+2)$ is the required equation of Parabola.

(xiv) $A(-1, 3, 2), B(-4, 2, -2), C(5, \lambda, \mu)$

If $O(0,0,0)$ be the reference point, then

$$p.v \text{ of } A = \overrightarrow{OA} = [-1, 3, 2]$$

$$p.v \text{ of } B = \overrightarrow{OB} = [-4, 2, -2]$$

$$p.v \text{ of } C = \overrightarrow{OC} = [5, \lambda, \mu]$$

$$\text{Taking } \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [-4, 2, -2] - [-1, 3, 2] = [-3, -1, -4]$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [5, \lambda, \mu] - [-4, 2, -2] = [9, \lambda - 2, \mu + 2]$$

If A, B and C are collinear, then $\overrightarrow{AB} = t \overrightarrow{BC}$ where t is scalar

$$[-3, -1, -4] = t[9, \lambda - 2, \mu + 2]$$

$$[-3, -1, -4] = [9t, (\lambda - 2)t, (\mu + 2)t]$$

$$-3 = 9t \Rightarrow t = -\frac{1}{3}$$

$$-1 = t(\lambda - 2) \Rightarrow -1 = -\frac{1}{3}(\lambda - 2) \Rightarrow \lambda = 5 \quad \because t = -\frac{1}{3}$$

$$-4 = t(\mu + 2) \Rightarrow -4 = -\frac{1}{3}(\mu + 2) \Rightarrow \mu = 10 \quad \because t = -\frac{1}{3}$$

(xv) $A(1,7,2), B(3,3,4), C(2,5,1)$

If $O(0,0,0)$ be the reference point, then

$$p.v \text{ of } A = \overrightarrow{OA} = [1, 7, 2]$$

$$p.v \text{ of } B = \overrightarrow{OB} = [3, 3, 4]$$

$$p.v \text{ of } C = \overrightarrow{OC} = [2, 5, 1]$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [3, 3, 4] - [1, 7, 2] = [2, -4, 2]$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [2, 5, 1] - [3, 3, 4] = [-1, 2, -3]$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = [2, -4, 2] \cdot [-1, 2, -3] = 2(-1) - 4(2) + 2(-3) = -12$$

$$|\overrightarrow{AB}| = \sqrt{(2)^2 + (-4)^2 + (2)^2} = \sqrt{24}$$

$$|\overrightarrow{BC}| = \sqrt{(-1)^2 + (2)^2 + (-3)^2} = \sqrt{14}$$

If θ be the angle between \overrightarrow{AB} and \overrightarrow{BC} , then

$$\cos \theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}| |\overrightarrow{BC}|} = \frac{-12}{\sqrt{24}\sqrt{14}}$$

$$\theta = 130.54$$

(xvi) $\underline{a} = [2, -1, 1], \underline{b} = [1, -3, -5]$

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 1 \\ 1 & -3 & -5 \end{vmatrix} = 8\underline{i} - 11\underline{j} - 5\underline{k}$$

$$|\underline{a} \times \underline{b}| = \sqrt{64 + 121 + 25} = \sqrt{210}$$

$$\text{Unit vector} = \frac{\underline{a} \times \underline{b}}{|\underline{a} \times \underline{b}|} = \frac{8\underline{i} - 11\underline{j} - 5\underline{k}}{\sqrt{210}}$$

$$\text{Required vector} = 7 \left(\frac{8\underline{i} - 11\underline{j} - 5\underline{k}}{\sqrt{210}} \right) = \frac{56}{\sqrt{210}}\underline{i} - \frac{77}{\sqrt{210}}\underline{j} - \frac{35}{\sqrt{210}}$$

SECTION C

Q3. $g(x) = \begin{cases} lx + 5 & \text{if } -5 < x < -2 \\ mx^2 - 2 & \text{if } x = -2 \\ x^3 - 5 & \text{if } x > -2 \end{cases}$ is continuous $\forall x$.

Value of the Function

at $x = -2$, $g(x) = mx^2 - 2$

$$g(-2) = m(-2)^2 - 2 = 4m - 2$$

Limit of the Function

L.H.Limit: $\lim_{x \rightarrow -2^-} g(x) = \lim_{x \rightarrow -2^-} (lx + 5) = -2l + 5$

R.H.Limit: $\lim_{x \rightarrow -2^+} g(x) = \lim_{x \rightarrow -2^+} (x^3 - 5) = (-2)^3 - 5 = -13$

Since $g(x)$ is continuous $\forall x$, so $\lim_{x \rightarrow -2} g(x)$ exists.

$$\lim_{x \rightarrow -2^-} g(x) = \lim_{x \rightarrow -2^+} g(x)$$

$$-2l + 5 = -13 \Rightarrow l = 9$$

Value = Limit

$$g(-2) = \lim_{x \rightarrow -2} g(x)$$

$$4m - 2 = -13$$

$$m = -\frac{11}{4}$$

Q4.

$$y = (\sin^{-1} x)^2$$

Differentiating w.r.t. x

$$y_1 = 2(\sin^{-1} x) \frac{d}{dx} (\sin^{-1} x)$$

$$y_1 = 2 \sin^{-1} x \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} y_1 = 2 \sin^{-1} x$$

Squaring both sides

$$(1-x^2)y_1^2 = 4(\sin^{-1} x)^2$$

$$(1-x^2)y_1^2 = 4y \quad \because y = (\sin^{-1} x)^2$$

Differentiating w.r.t. x

$$(1-x^2)2y_1y_2 - 2xy_1^2 = 4y_1$$

$$(1-x^2)2y_1y_2 - 2xy_1^2 - 4y_1 = 0$$

$$2y_1[(1-x^2)y_2 - xy_1 - 2] = 0$$

$$y_1[(1-x^2)y_2 - xy_1 - 2] = 0$$

$$(1-x^2)y_2 - xy_1 - 2 \text{ where as } y_1 \neq 0$$

Differentiating w.r.t. x

$$(1-x^2)y_3 - 2xy_2 - xy_2 - y_1 = 0$$

$$(1-x^2)y_3 - 3xy_2 - y_1 = 0$$

Q5.

$$\text{Let } a = 2t + 4$$

$$\frac{dv}{dt} = 2t + 4$$

$$\int dv = \int (2t + 4) dt$$

$$v = 2\left(\frac{t^2}{2}\right) + 4t + C_1$$

$$v = t^2 + 4t + C_1 \rightarrow \text{Eqn}(i)$$

$$\text{Initially } v(0) = 20 \text{ms}^{-1}$$

$$20 = 0 + 0 + C_1 \Rightarrow C_1 = 20$$

$$\text{Eqn}(i) \rightarrow v = (t^2 + 4t + 20) \text{ms}^{-1}$$

$$\frac{ds}{dt} = t^2 + 4t + 20$$

$$\int ds = \int (t^2 + 4t + 20) dt$$

$$s = \frac{t^3}{3} + 4\left(\frac{t^2}{2}\right) + 20t + C_2$$

$$s = \frac{t^3}{3} + 2t^2 + 20t + C_2 \rightarrow \text{Eqn(ii)}$$

Initially $s(0) = 0$

$$0 = 0 + 0 + 0 + C_2 \Rightarrow C_2 = 0$$

$$\text{Eqn(ii)} \rightarrow s = \left(\frac{t^3}{3} + 2t^2 + 20t\right)m$$

Q6 Let $l_1 : 5x + 6y = 30$ be the given line

(a) l_1 cuts x -axis at $P(x, 0)$ giving $5x + 6(0) = 30 \Rightarrow x = 6 \Rightarrow P(6, 0)$

l_1 cuts y -axis at $Q(0, y)$ giving $5(0)x + 6y = 30 \Rightarrow y = 5 \Rightarrow Q(0, 5)$

(b) $|\overline{PQ}| = \sqrt{(0-6)^2 + (5-0)^2} = \sqrt{36+25} = \sqrt{61}$

(c) Here R is the mid-point of \overline{PQ}

$$R\left(\frac{6+0}{2}, \frac{0+5}{2}\right) \Rightarrow R\left(3, \frac{5}{2}\right)$$

(d) Equation of \overline{OR} with $O(0,0) = (x_1, y_1)$ and $R\left(3, \frac{5}{2}\right) = (x_2, y_2)$

in two points form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{\frac{5}{2} - 0}{3 - 0}(x - 3)$$

$$3y = \frac{5}{2}(x - 3)$$

$$5x - 6y - 15 = 0$$

Q7. Let Saira bought x number of bananas and y number of apples.

Price of one banana is Rs.6 and price of one apple is Rs.10

(a) According to the given conditions

$x \geq 1$, $y \geq 1$, $x + y \leq 5$ are the three constraints(inequalities).

(b) Graph is shown

(c) Shaded region is a triangle in the first quadrant with vertices as corner points

$A(1,1)$, $B(4,1)$ and $C(1,4)$

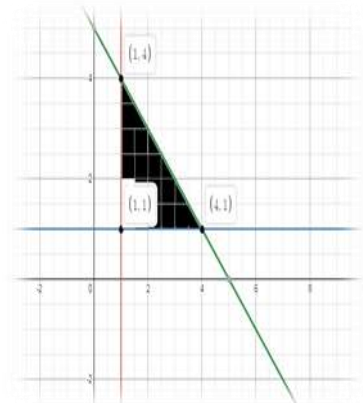
(d) Profit on one banana = Rs. 26

Profit on x bananas = Rs. $26x$

Profit on one apple = Rs. 10

Profit on y apples = Rs. $10y$

Objective Function: $P(x, y) = 26x + 10y$



Corner points	$P(x, y) = 26x + 10y$
A(1,1)	$P(1,1) = 26(1) + 10(1) = 36$
B(4,1)	$P(4,1) = 26(4) + 10(1) = 114$
C(1,4)	$P(1,4) = 26(1) + 10(4) = 66$

Hence to give shopkeeper maximum profit, Saira must buy 4 bananas and one apple.
The maximum profit to the shopkeeper is Rs 114

Q8. Let $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ be the required vertical Hyperbola.

Given that: *Sum of the axis* = 32

$$2a + 2b = 32$$

$$a + b = 16 \text{ -----eqn-I}$$

Given that: eccentricity $e = \sqrt{7}$

$$\therefore b^2 = a^2(e^2 - 1)$$

$$b^2 = a^2((\sqrt{7})^2 - 1)$$

$$b^2 = 6a^2$$

$$b = \sqrt{6}a \text{ -----eqn-II}$$

Substituting b's value in en-I

$$a + \sqrt{6}a = 16$$

$$a = 4.64$$

Substituting a's value in en-II

$$b = \sqrt{6}(4.64) = 11.37$$

Required Hyperbola: $\frac{x^2}{4.64^2} - \frac{y^2}{11.37^2} = 1$

Equation of the asymptotes: $y = \pm \frac{a}{b}x$

$$y = \frac{4.64}{11.37}x$$