



Model Question Paper (3rd Set Solution)
Mathematics HSSC-I

Section – A

1	B	2	B	3	C	4	B	5	B	6	A	7	A	8	C	9	C	10	A
11	B	12	B	13	D	14	B	15	B	16	C	17	A	18	B	19	B	20	D

SECTION-B

Q2.

(i) $z = \frac{5-3i}{1+i} \times \frac{1-i}{1+i} = \frac{5-8i+3}{1-(-1)} = 1 - 4i$

a) Modulus of $z = |z| = \sqrt{(1)^2 + (-4)^2} = \sqrt{17}$

b) Argument of $z = \tan\theta = -\frac{4}{1} \Rightarrow \theta = 275.60^\circ$

(ii) **Closure Property:**

Let x, y be any two elements of G such that $x = a + \sqrt{5}b, y = c + \sqrt{5}d \quad \forall a, b, c, d \in Q$

$$x + y = (a + \sqrt{5}b) + (c + \sqrt{5}d) = (a + c) + \sqrt{5}(b + d)$$

Since $a + c$ and $b + d$ are element of Q

$$\Rightarrow [(a + c) + \sqrt{5}(b + d)] \in G$$

$\therefore G$ is closed with respect to $' + '$.

Associative Property:

Let $(a + \sqrt{5}b), (c + \sqrt{5}d), (e + \sqrt{5}f) \in G, \quad \forall a, b, c, d, e, f \in Q$

$$\text{L.H.S} = [(a + \sqrt{5}b) + (c + \sqrt{5}d)] + (e + \sqrt{5}f)$$

$$= [(a + c) + \sqrt{5}(b + d)] + (e + \sqrt{5}f)$$

$$= (a + c + e) + \sqrt{5}(b + d + f) \quad \rightarrow \text{eqn(1)}$$

$$\text{R.H.S} = (a + \sqrt{5}b) + [(c + \sqrt{5}d) + (e + \sqrt{5}f)]$$

$$= (a + \sqrt{5}b) + [(c + e) + \sqrt{5}(d + f)]$$

$$= (a + c + e) + \sqrt{5}(b + d + f) \quad \rightarrow \text{eqn(2)}$$

From Equations (1) & (2), G is Associative with respect to $' + '$.

Existence of Identity:

$\forall (a + \sqrt{5}b) \in G$, there exist $(0 + \sqrt{5}0) \in G$ such that

$$[(a + \sqrt{5}b) + (0 + \sqrt{5}0)] = [(0 + \sqrt{5}0) + (a + \sqrt{5}b)] = (a + \sqrt{5}b)$$

$\therefore (0 + \sqrt{5}0)$ is an identity element in G with respect to $' + '$

Existence of Inverse:

For each $(a + \sqrt{5}b) \in G$ there exist $(-a - \sqrt{5}b) \in G$

$$(a + \sqrt{5}b) + [(-a) + \sqrt{5}(-b)] = [a + (-a)] + \sqrt{5}[(-b) + b] = 0 + \sqrt{5}0 = \text{Identity}$$

$\therefore (-a) + \sqrt{5}(-b)$ is the inverse of $a + \sqrt{5}b$

Hence G is a group with respect to $' + '$.

$$(iii) \begin{vmatrix} 4 & 4 & 4 & m \\ 4 & 4 & m & 4 \\ 4 & m & 4 & 4 \\ m & 4 & 4 & 4 \end{vmatrix} = 0$$

$$\begin{vmatrix} 4 & 4 & 4 & m + 12 \\ 4 & 4 & m & m + 12 \\ 4 & m & 4 & m + 12 \\ m & 4 & 4 & m + 12 \end{vmatrix} = 0 \quad \text{Add } C_1, C_2, C_3 \text{ in } C_4$$

$$(m + 12) \begin{vmatrix} 4 & 4 & 4 & 1 \\ 4 & 4 & m & 1 \\ 4 & m & 4 & 1 \\ m & 4 & 4 & 1 \end{vmatrix} = 0 \quad \text{Take } (m + 12) \text{ common from } C_4$$

$$R_2 - R_1, R_3 - R_1, R_4 - R_1$$

$$(m + 12) \begin{vmatrix} 4 & 4 & 4 & 1 \\ 0 & 0 & m - 4 & 0 \\ 0 & m - 4 & 0 & 0 \\ m - 4 & 0 & 0 & 0 \end{vmatrix} = 0$$

$$(m + 12)[(m - 4)^3] = 0$$

(Product of the principle diagonal elements in a triangular matrix)

$$\text{Either } (m + 12) = 0 \text{ or } (m - 4)^3 = 0$$

$$m = -12, 4$$

$$(iv) \text{ Let } P(x) = x^3 - 8sx^2 - 4tx + 9$$

As -2 and 2 are the roots of $P(x)$

$$\text{So } P(-2) = 0 \text{ and } P(2) = 0$$

$$\therefore P(-2) = (-2)^3 - 8s(-2)^2 - 4t(-2) + 9 = 0$$

$$-32s + 8t + 1 = 0$$

$$32s - 8t = 1 \quad \rightarrow \text{eqn(1)}$$

$$\text{Now } P(2) = (2)^3 - 8s(2)^2 - 4t(2) + 9 = 0$$

$$-32s - 8t + 17 = 0$$

$$32s + 8t = 17 \quad \rightarrow \text{eqn(2)}$$

On adding Equations (1) & (2), we get $s = \frac{9}{32}$

On subtracting Equations (1) from (2), we get $t = 1$

(v) If α, β are the roots of $x^2 - 3x + 2 = 0$ so $\alpha + \beta = 3$ & $\alpha\beta = 2$

If roots are $\left(1 + \frac{3}{\alpha}\right)$ and $\left(1 + \frac{3}{\beta}\right)$, then

$$\text{Sum of roots: } S = \left(1 + \frac{3}{\alpha}\right) + \left(1 + \frac{3}{\beta}\right) = 2 + 3\left(\frac{\alpha+\beta}{\alpha\beta}\right) = 2 + 3\left(\frac{3}{2}\right) = \frac{13}{2}$$

$$\text{Product of roots: } P = \left(1 + \frac{3}{\alpha}\right)\left(1 + \frac{3}{\beta}\right) = 1 + 3\left(\frac{\alpha+\beta}{\alpha\beta}\right) + \frac{9}{\alpha\beta} = 1 + 3\left(\frac{3}{2}\right) + \frac{9}{2} = 10$$

Required Equation: $x^2 - Sx + P = 0$

$$x^2 - \frac{13}{2}x + 10 = 0$$

$$2x^2 - 13x + 20 = 0$$

$$(vi) \frac{7-3x+x^2}{(3+x)(1-x)^2} = \frac{A}{(3+x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2} \quad \rightarrow \text{eqn(1)}$$

$$7 - 3x + x^2 = A(1-x)^2 + B(3+x)(1-x) + C(3+x) \quad \rightarrow \text{eqn(2)}$$

For A put $3+x=0$ or $x=-3$ in eqn(2)

$$7 - 3(-3) + (-3)^2 = A(1+3)^2 + 0 + 0 \quad \Rightarrow A = \frac{25}{16}$$

For C put $1-x=0$ or $x=1$ in eqn(2)

$$7 - 3(1) + (1)^2 = 0 + 0 + C(3+1) \quad \Rightarrow C = \frac{5}{4}$$

Simplifying eqn(2) as

$$7 - 3x - x^2 = A(1+x^2 - 2x) + B(-x^2 - 2x + 3) + C(3+x)$$

Equating the coefficients of x^2

$$-1 = A - B \quad \Rightarrow -1 = \frac{25}{16} - B \quad \Rightarrow B = \frac{9}{16}$$

Substituting the values of A, B and C in eqn (1)

$$\frac{7-3x+x^2}{(3+x)(1-x)^2} = \frac{25}{16(3+x)} + \frac{9}{16(1-x)} + \frac{5}{4(1-x)^2}$$

(vii) In A.P $a_1 = 74$ and $d = -6$

$$(a) S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_n = \frac{n}{2}[2(74) + (n-1)(-6)]$$

$$S_n = n(77 - 3n)$$

$$(b) S_n = n(77 - 3n)$$

$$380 = n(77 - 3n) \quad \because S_n = 380$$

$$3n^2 - 77n + 380 = 0$$

$$n = 19 \text{ or } \frac{20}{3}$$

$$\text{Since } n = \frac{20}{3} \notin \mathbb{N} \quad \Rightarrow \quad n = 19$$

(viii) G.P: $a_1, a_1r, a_1r^2, a_1r^3, \dots, a_1r^{n-1}, \dots$

Given that $a_2 = a_1r = \frac{1}{4} \Rightarrow a_1 = \frac{1}{4r} \rightarrow eqn(1)$

G.S: $a_1 + a_1r + a_1r^2 + a_1r^3 + \dots + a_1r^{n-1} + \dots$

Taking $S_\infty = \frac{a_1}{1-r}$

$2 = \frac{a_1}{1-r} \quad \because S_\infty = 2$

$2(1-r) = a_1$

Using eqn(1)

$2(1-r) = \frac{1}{4r}$

$8r^2 - 8r + 1 = 0$

$r = \frac{2 \pm \sqrt{2}}{4}$

Substituting it in eqn(1)

$a_1 = \frac{1}{2 \pm \sqrt{2}}$

(ix) Total arrangements = $\frac{7!}{3!2!1!1!} = 420$

Numbers less than 20000000 (when we put 0 at extreme left position) = $\frac{6!}{3!2!1!1!} = 60$

Numbers greater than 20000000 = $420 - 60 = 360$

(x) Sample space $n(S) = 52$

Let A be the event that the selected card is a heart card $\Rightarrow n(A) = 13$

Let B be the event that the selected card is a face card $\Rightarrow n(B) = 12$

$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52}$ and $P(B) = \frac{n(B)}{n(S)} = \frac{12}{52}$

$A \cap B = \{\text{king of heart, jack of heart, queen of heart}\} \Rightarrow n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{52}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$

(xi) $(1 - kx)^n = 1 - 10x + 60x^2 + \dots$

Comparing with

$(1 - kx)^n = 1 - nkx + \frac{n(n-1)}{2!} k^2 x^2 + \dots$

We get $nk = 10 \Rightarrow k = \frac{10}{n} \rightarrow eqn(1)$

and $\frac{n(n-1)}{2!} k^2 = 60 \Rightarrow \frac{n(n-1)}{2!} \left(\frac{10}{n}\right)^2 = 60$ using eqn(1)

$\frac{5(n-1)}{n} = 6 \Rightarrow n = -5$

$k = \frac{10}{-5} = -2$ Using eqn(1)

$$(xii) \quad \frac{24 \cos \theta - 5 \sin^2 \theta}{\cos^2 \theta + 5 \cos \theta} = 5 - \sec \theta$$

$$L.H.S = \frac{24 \cos \theta - 5 \sin^2 \theta}{\cos^2 \theta + 5 \cos \theta}$$

$$L.H.S = \frac{24 \cos \theta - 5(1 - \cos^2 \theta)}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{5 \cos^2 \theta + 24 \cos \theta - 5}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{5 \cos^2 \theta + 25 \cos \theta - \cos \theta - 5}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{(5 \cos \theta - 1)(\cos \theta + 5)}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{5 \cos \theta}{\cos \theta} - \frac{1}{\cos \theta} = 5 - \sec \theta$$

$$(xiii) \quad \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$$

$$L.H.S = \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x}$$

$$L.H.S = \frac{1 - (1 - 2 \sin^2 x) + 2 \sin x \cos x}{1 + (1 - 2 \sin^2 x) + 2 \sin x \cos x}$$

$$L.H.S = \frac{2 \sin^2 x + 2 \sin x \cos x}{2(1 - \sin^2 x) + 2 \sin x \cos x}$$

$$L.H.S = \frac{2 \sin x (\sin x + \cos x)}{2 \cos^2 x + 2 \sin x \cos x}$$

$$L.H.S = \frac{2 \sin x (\sin x + \cos x)}{2 \cos x (\cos x + \sin x)}$$

$$L.H.S = \tan x$$

$$L.H.S = R.H.S$$

$$(xiv) \text{ Let } a_1 = a + d, b_1 = a + 2d, c_1 = a + 3d$$

$$\cos \gamma = \frac{a_1^2 + b_1^2 - c_1^2}{2 a_1 b_1}$$

$$\cos \gamma = \frac{(a + d)^2 + (a + 2d)^2 - (a + 3d)^2}{2 (a + d)(a + 2d)} \quad \text{substituting the values}$$

$$\cos \gamma = \frac{a^2 + d^2 + 2ad + a^2 + 4d^2 + 4ad - a^2 - 9d^2 - 6ad}{2 (a + d)(a + 2d)}$$

$$\cos \gamma = \frac{a^2 - 4d^2}{2 (a + d)(a + 2d)} = \frac{(a - 2d)(a + 2d)}{2(a + d)(a + 2d)} = \frac{a - 2d}{2(a + d)} = \frac{a}{2(a + d)} - \frac{d}{(a + d)}$$

$$\begin{aligned}
\text{(xv)} \quad & \tan^{-1}\left(\frac{1+\frac{x}{2}}{1-\frac{x}{2}}\right) + \tan^{-1}\left(\frac{x}{2}\right) \\
& = \tan^{-1}\left(\frac{2+x}{2-x}\right) + \tan^{-1}\left(\frac{x}{2}\right) \\
& = \tan^{-1}\left(\frac{\frac{2+x}{2-x} + \frac{x}{2}}{1 - \left(\frac{2+x}{2-x}\right)\left(\frac{x}{2}\right)}\right) \quad \because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right) \\
& = \tan^{-1}\left(\frac{2(2+x) + x(2-x)}{2(2-x) - x(2+x)}\right) \\
& = \tan^{-1}\left(\frac{4 + 4x - x^2}{4 - 4x - x^2}\right)
\end{aligned}$$

$$\begin{aligned}
\text{(xvi)} \quad & 2 \cos 2\theta = -3 - 4\cos\theta \\
& 2(2 \cos^2 \theta - 1) + 3 + 4\cos\theta = 0 \\
& 4 \cos^2 \theta + 4\cos\theta + 1 = 0 \\
& (2\cos\theta + 1)^2 = 0 \\
& 2\cos\theta + 1 = 0 \\
& \cos\theta = -\frac{1}{2}
\end{aligned}$$

Here $\cos\theta < 0 \Rightarrow \theta$ lies in Quadrants II & III with reference angle $\frac{\pi}{3}$

$$\text{In Quad-II: } \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\text{In Quad-III: } \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\text{Solution Set} = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

SECTION-C

$$\text{Q3. } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Using Row Operations

$$\begin{aligned}
[A : I] &= \begin{bmatrix} 1 & 2 & 3 & : & 1 & 0 & 0 \\ 2 & 5 & 3 & : & 0 & 1 & 1 \\ 1 & 0 & 8 & : & 0 & 0 & 1 \end{bmatrix} \\
R \sim \begin{bmatrix} 1 & 2 & 3 & : & 1 & 0 & 0 \\ 0 & 1 & -3 & : & -2 & 1 & 0 \\ 0 & -2 & 5 & : & -1 & 0 & 1 \end{bmatrix} & \quad R_2 - 2R_1 ; R_3 - R_1 \\
R \sim \begin{bmatrix} 1 & 0 & 9 & : & 5 & -2 & 0 \\ 0 & 1 & -3 & : & -2 & 1 & 0 \\ 0 & 0 & -1 & : & -5 & 2 & 1 \end{bmatrix} & \quad R_1 - 2R_2 ; R_3 + 2R_2 \\
R \sim \begin{bmatrix} 1 & 0 & 9 & : & 5 & -2 & 0 \\ 0 & 1 & -3 & : & -2 & 1 & 0 \\ 0 & 0 & 1 & : & 5 & -2 & -1 \end{bmatrix} & \quad (-1)R_3 \\
R \sim \begin{bmatrix} 1 & 0 & 0 & : & -40 & 16 & 9 \\ 0 & 1 & 0 & : & 13 & -5 & -3 \\ 0 & 0 & 1 & : & 5 & -2 & -1 \end{bmatrix} & \quad R_1 - 9R_3 ; R_2 + 3R_3
\end{aligned}$$

$$= [I : A^{-1}]$$

$$\text{Hence } A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Q4. (a) Number of men = 6, Number of women = 4

- (i) Number of ways that a 5 member Committee including exactly 3 men = $\binom{6}{3} \times \binom{4}{2} = 120$
(ii) Number of ways that a 5 member Committee including at least 2 women
 $= \binom{6}{3} \times \binom{4}{2} + \binom{6}{2} \times \binom{4}{3} + \binom{6}{1} \times \binom{4}{4} = 120 + 60 + 6 = 166$
- (b) $S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

$$\Rightarrow n(S) = 12$$

Since A be the event that a Tail appear in tossing a coin.

$$A = \{(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\} \Rightarrow n(A) = 6$$

Since B be the event that 4 dots appear in rolling a dice.

$$B = \{(H, 4), (T, 4)\} \Rightarrow n(B) = 2$$

$$A \cap B = \{(T, 4)\} \Rightarrow n(A \cap B) = 1$$

$$P(A) = \frac{6}{12} = \frac{1}{2}, \quad P(B) = \frac{2}{12} = \frac{1}{6}, \quad P(A \cap B) = \frac{1}{12}$$

For Independent Events A and B

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{1}{12} = \left(\frac{1}{2}\right) \left(\frac{1}{6}\right)$$

$$\frac{1}{12} = \frac{1}{12}$$

Hence A and B are the independent events.

Q5. $\left(\frac{3y^2}{2} - \frac{1}{3y}\right)^6$

$$T_{r+1} = \binom{n}{r} (a)^{n-r} (b)^r \quad \text{where } a = \frac{3y^2}{2}, b = -\frac{1}{3y}, n = 6$$

$$T_{r+1} = \binom{6}{r} \left(\frac{3y^2}{2}\right)^{6-r} \left(-\frac{1}{3y}\right)^r$$

$$T_{r+1} = \binom{6}{r} \frac{(3)^{6-r} y^{2(6-r)} (-1)^r}{(2)^{6-r} (3)^r (y)^r}$$

$$T_{r+1} = \binom{6}{r} \frac{(-1)^r (3)^{6-2r} (y)^{12-3r}}{(2)^{6-r}}$$

- (i) For the term involving y^3 put $12 - 3r = 3 \Rightarrow r = 3$

$$T_{3+1} = \binom{6}{3} \frac{(-1)^3 (3)^{6-6} (y)^{12-9}}{(2)^{6-3}} = -\frac{5}{2} y^3$$

- (ii) For the term independent of y put $12 - 3r = 0 \Rightarrow r = 4$

$$T_{4+1} = \binom{6}{4} \frac{(-1)^4 (3)^{6-8} (y)^{12-12}}{(2)^{6-4}} = \frac{5}{12}$$

(iii) Middle term = $\binom{n+2}{2} th \text{ term}$

Middle term = $\binom{6+2}{2} th = 4th \text{ term}$

$$T_{r+1} = \binom{6}{r} \frac{(-1)^r (3)^{6-2r} (y)^{12-3r}}{(2)^{6-r}}$$

$$T_{3+1} = \binom{6}{3} \frac{(-1)^3 (3)^{6-6} (y)^{12-9}}{(2)^{6-3}} = -\frac{5}{2} y^3$$

Q6. Statement: For any two angles α and β (real numbers) then,

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Proof:

Consider four points A, B, C and D on a unit circle with center at O

Such that $Arc \widehat{AB} = Arc \widehat{CD}$ and $Chord \overline{AB} = Chord \overline{CD}$

Let $\angle AOD = \alpha$ and $\angle AOC = \beta$

Then $\angle AOB = \alpha - \beta$, $\angle COD = \alpha - \beta$

Here α, β and $\alpha - \beta$ are in standard position.

Co-ordinates of point A are (1,0)

Co-ordinates of point B are $(\cos(\alpha - \beta), \sin(\alpha - \beta))$

Co-ordinates of point C are $(\cos \beta, \sin \beta)$

Co-ordinate of point D are $(\cos \alpha, \sin \alpha)$

Since $|AB| = |CD| \quad \because Chord \overline{AB} = Chord \overline{CD}$

$$|AB|^2 = |CD|^2$$

$$[\cos(\alpha - \beta) - 1]^2 + [\sin(\alpha - \beta) - 0]^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$$

$$\cos^2(\alpha - \beta) - 2\cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta) = \cos^2 \alpha + \cos^2 \beta - 2\cos \alpha \cos \beta$$

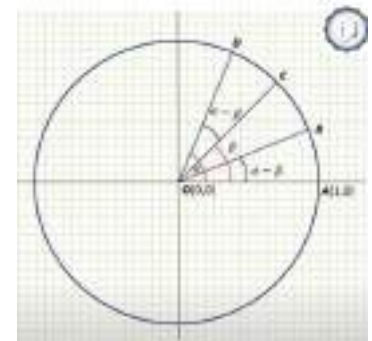
$$+ \sin^2 \alpha + \sin^2 \beta - 2\sin \alpha \sin \beta$$

$$[\cos^2(\alpha - \beta) + \sin^2(\alpha - \beta)] - 2\cos(\alpha - \beta) + 1 = (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta)$$

$$-2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$1 - 2\cos(\alpha - \beta) + 1 = 1 + 1 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$



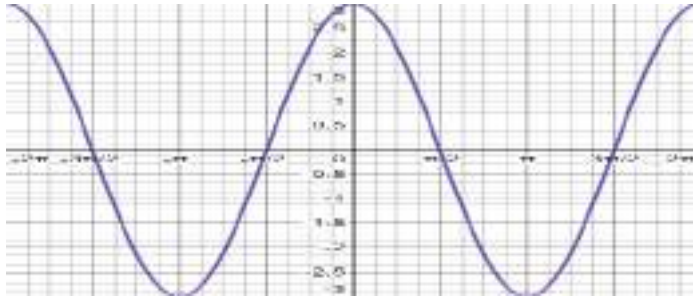
Q7. $y = 3 \cos x$ for $-2\pi \leq x \leq 2\pi$

(a) $3 \cos x = 3 \cos(x + 2\pi) \quad \because$ Period of cosine function is 2π

Hence 2π is the period of $3 \cos x$

(b) Table of values

x	0°	$\pm 30^\circ$	$\pm 60^\circ$	$\pm 90^\circ$	$\pm 120^\circ$	$\pm 150^\circ$	$\pm 180^\circ$
y	3	2.6	1.5	0	-1.5	-2.6	-3
x	$\pm 210^\circ$	$\pm 240^\circ$	$\pm 270^\circ$	$\pm 300^\circ$	$\pm 330^\circ$	$\pm 360^\circ$	$\pm 210^\circ$
y	-2.6	-1.5	0	1.5	2.6	3	-2.6



Q8. $\cos^{-1}\left(\frac{3}{5}\right) - 2 \tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\left(\frac{117}{125}\right)$

Proof: L.H.S = $\cos^{-1}\left(\frac{3}{5}\right) - 2 \tan^{-1}\left(\frac{3}{4}\right)$

L.H.S = $\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right) \quad \because 2 \tan^{-1} A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

L.H.S = $\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{24}{7}\right) \quad \rightarrow \text{eqn(1)}$

Let $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$ iff $\cos \alpha = \frac{3}{5} > 0$ in $[0, \pi] \Rightarrow \alpha \in \text{Quad - I}$

i.e. $\sin \alpha = +\sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

Let $\beta = \tan^{-1}\left(\frac{24}{7}\right)$ iff $\tan \beta = \frac{24}{7} > 0$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \beta \in \text{Quad - I}$

$\sec \beta = +\sqrt{1 + \tan^2 \beta} = \sqrt{1 + \frac{576}{49}} = \frac{25}{7}$

$\sec \beta = \frac{25}{7} \Rightarrow \cos \beta = \frac{7}{25}$

$\sin \beta = +\sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{49}{625}} = \frac{24}{25}$

$\because \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$\cos(\alpha - \beta) = \left(\frac{3}{5}\right)\left(\frac{7}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{24}{25}\right)$

$\cos(\alpha - \beta) = \frac{117}{125}$

$\alpha - \beta = \cos^{-1}\frac{117}{125}$

$\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{24}{7}\right) = \cos^{-1}\frac{117}{125}$

$\cos^{-1}\left(\frac{3}{5}\right) - 2 \tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\frac{117}{125}$